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## GEOMETRICAL ILLUSTRATIONS OF NEWLANDS' AND MENDELEJEFF'S PERIODIC LAW OF THE ATOMIC WEIGHTS OF THE CHEMICAL ELEMENTS.

By the REV. SAMUEL HAUGHTON, M.D. (Dubl. et Bonon.), D.C.L. (Oxon.), LL.D. (Cantab. et Edin.), President of the Royal Irish Academy.

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# VII.

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[Read April 28, 1888.]

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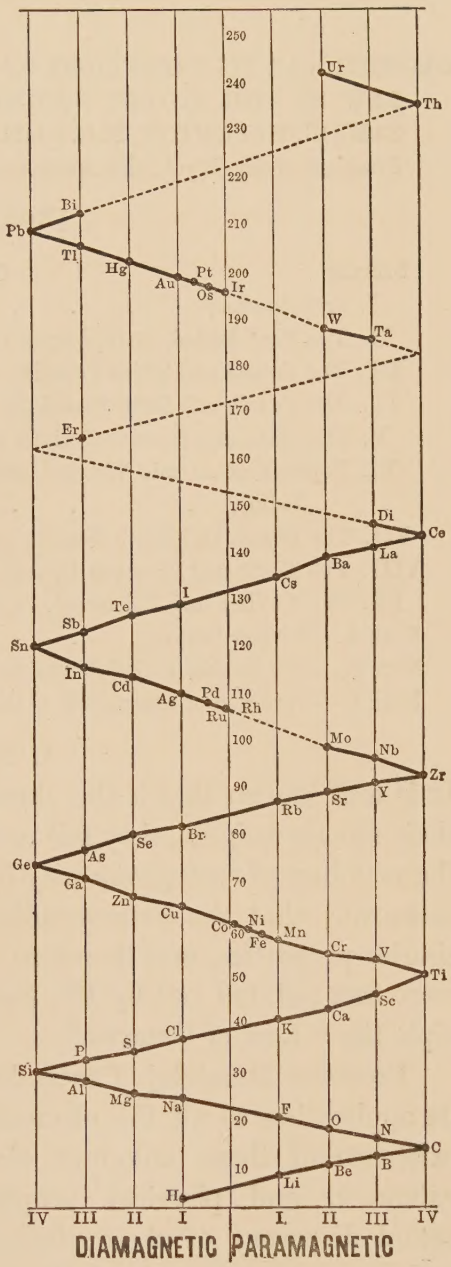
## I. Introduction.

It is well known that if the chemical elements be arranged in the order of their atomic weights, they fall naturally into groups of seven each, and that the members of each group vary in properties with increased atomic weight; moreover, that the corresponding members of successive groups possess similar properties, and therefore are periodically related. These relations were first pointed out by Mr. Newlands, who termed the underlying principle the "Law of Octaves."

Professor Mendelejeff extended the Law of Periodicity so as to render its application to all the elements highly probable, and he predicted the existence of three unknown elements, assigning their atomic weights, valencies, and physical properties, approximately. The subsequent actual discovery of these elements, now known as Gallium, Scandium, and Germanium, attracted the attention of mathematicians as well as that

of chemists; for a science which can foretell the existence and properties of unknown bodies, which are afterwards found, with the properties predicted of them, must rest essentially on the Laws of Number and Space.




Before the discovery of two out of the three above-named predicted elements, Professor E. Reynolds showed that their atomic weights and chief properties could be arrived at by construction when most of the elements whose atomic weights and relations are sufficiently well known were plotted on the annexed diagram, which also brings clearly into view the fact that the members of the alternate periods are those which are most closely related. The predicted and observed positions of Scandium and Germanium were found to very nearly coincide.

In this diagram the elements are plotted according to their atomic weights and valencies; the vertical co-ordinates being atomic weights, and the horizontal co-ordinates being the most characteristic valencies of the elements, as Professor Reynolds pointed out that in each period the valency regularly increases with the gain in atomic weight up to the middle term, and then less regularly decreases to the seventh member—the first three elements being chiefly electro-positive, and the last three generally electro-negative.

For the purpose of mathematical discussion it will be better to take the periods in pairs or double octaves.

# VII.

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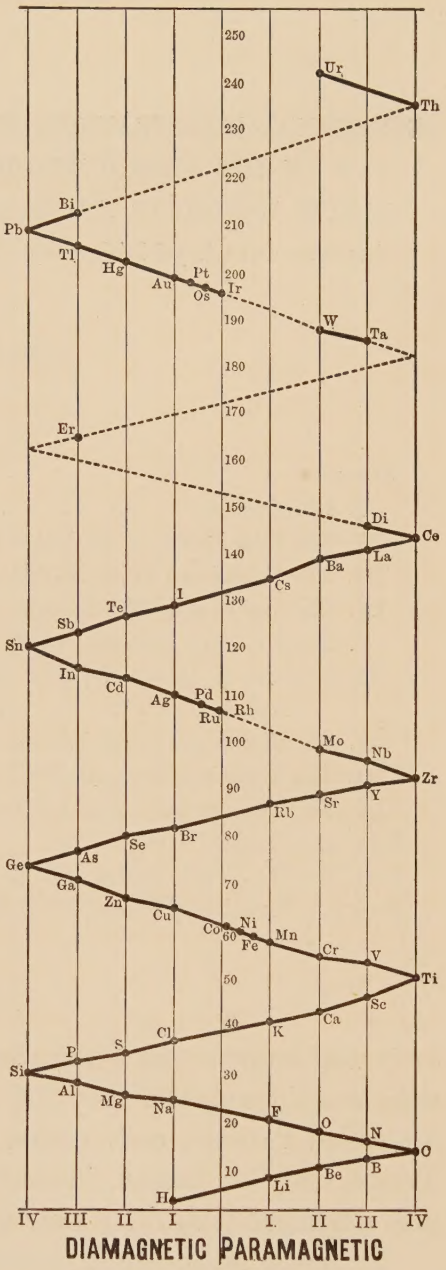
## I. Introduction.

It is well known that if the chemical elements be arranged in the order of their atomic weights, they fall naturally into groups of seven each, which are called Periods, each period consisting of two Monads, two Dyads, two Triads, and one Tetrad. This fact was first stated by Professor Newlands, who described it as the "Law of Octaves."

Professor Mendelejeff extended the Law of Periodicity so as to render its application to all the elements highly probable, and he predicted the

existence of three unknown elements, assigning their atomic weights, valencies, and physical properties, approximately. The subsequent actual discovery of these elements, now known as Gallium, Scandium, and Germanium, attracted the attention of mathematicians as well as that of chemists; for a science which can foretell the existence and properties of unknown bodies, which are afterwards found, with the properties predicted of them, must rest essentially on the Laws of Number and Space.

Professor Reynolds further showed that the alternate periods of seven resembled each other more closely in physical properties than the successive periods. Thus the second, fourth, sixth, &c., periods are paramagnetic, while the first, third, fifth, &c., are diamagnetic. Hence it follows that, for the purpose of mathematical discussion, it will be better to take the periods in pairs, or double octaves. The accompanying figure (1), which has been published by Professor Reynolds, gives at a glance a complete view of all the known elements, with their chemical and physical properties.




In this diagram the elements are plotted according to their atomic weights and valencies; the vertical co-ordinates being atomic weights, and the horizontal co-ordinates being valencies, counted (for convenience of plotting) positive or negative, according as the element belongs to an odd or even period of seven.

The first attempt I made to apply geometry to Dr. Reynolds' representation of the Periodic Law was by means of Fourier's Theorem; thus:

Let $v$ denote the valency of any element, and let $\phi = n(t - t_0)$, we have

$$v = a_0 + a_1 \cos\phi + a_2 \cos 2\phi + \&c., + a_n \cos n\phi$$
$$+ b_1 \sin\phi + b_2 \sin 2\phi + \&c., + b_n \sin n\phi, \qquad (1)$$

where $t$ denotes the atomic weight, and $n$, $t_0$, $a_0$, $a_1$, $b_1$, &c., $a_n$, $b_n$ are coefficients, not necessarily constant, but probably functions of $t$, to be afterwards determined. $\phi$ depends upon $n$ and $t_0$, which are arbitrary, but may be so chosen as to facilitate the numerical calculations.

Fourier's Theorem has proved itself to be a simple and powerful method of interpreting the periodic laws of tidal and meteorological phenomena, because these periodic laws involve only a few terms, and the observations can be made at equal intervals; but a very few trials soon showed me that the periodic laws of chemistry fulfil neither of these conditions. In order to obtain even a moderate degree of approximation to the known valencies, we must carry on the calculation to terms of a high order; so high indeed as to suggest the suspicion that, in order to be exact, the number of terms used must be practically infinite.

Since $\phi$ is arbitrary, an infinite number of Fourier curves may be drawn through a finite number of points, and when the number of terms is great, these curves have little more value than so many lines drawn with a free hand through those points. I was, therefore, deterred from discussing the periodic law of chemistry, by means of Fourier's Theorem, from the consideration of the great number of terms necessary for an approximation, and from the fact that the atomic weights are not given at equal intervals, but irregularly—a circumstance which increases seriously the difficulty of calculating the coefficients.*

Another consideration, however, weighed on my mind much more than the difficulty of the calculations, and that was the certain fact, that there are nine well-known elements which cannot be brought either upon

* *Vide* Note A, p. 242.

Dr. Reynolds' zigzag line, or any possible interpretation of that line by the Fourier Function. These elements are—

| | | Atomic Weight. |
|---|---|---|
| *Iron group*, | 1. Iron, | 56 |
| | 2. Nickel, | 58·5 |
| | 3. Cobalt, | 59 |
| *Palladium group*, | 4. Rhodium, | 104 |
| | 5. Ruthenium, | 104·5 |
| | 6. Palladium, | 106 |
| *Platinum group*, | 7. Osmium, | 191·1 |
| | 8. Iridium, | 192·5 |
| | 9. Platinum, | 194·3 |

No geometrical interpretation of the Chemical Periodic Law can be considered complete which does not place these nine elements upon the general curve which is to represent the complete Periodic Law.

The geometrical principles which I have adopted in place of the Fourier curves possess the following important advantages :—

1°. The curves drawn by me pass accurately through all the given points, and admit of no successive approximations.

2°. They constitute the simplest curves that can be drawn through the points; they are unique, and allow no rival curves.

3°. The nine elements which find no place on Dr. Reynolds' diagram or on the Fourier curves, settle quietly down in their natural positions on my curves, although those curves are drawn from data supplied by the other elements, excluding all consideration of the aberrant nine.

Each double period of elements gives a collocation of fourteen points depending on atomic weights and valencies.

A unique quartic curve can be drawn accurately through these fourteen points, and all other curves drawn through the fourteen points must be of a higher order.

I find that the unique quartic curves passing through the successive fourteen points belong to the class of quartics having two real and two imaginary asymptotes, and that some of them reduce to a straight line and cubic with one real asymptote.

All the quartic curves are connected together by laws of variation of parameters, which I have investigated, and these laws enable us to draw the complete curves in cases in which the whole fourteen elements are not known. The periods containing tantalum, wolfram, and gold, mercury, thallium, lead, and bismuth, afford an example of this calculation.

## II. The First Period; or Hydrogen Period.

All that is known of this period is the very important element hydrogen, respecting which I adopt Dr. Reynolds' view, that it is the seventh and last of a period respecting the other six elements of which nothing is known. Mendelejeff supposed that the six companions of hydrogen might be sought for between hydrogen and lithium, but Dr. Reynolds' curve indicates clearly that they lie before hydrogen. At the close of this Paper I shall show that the central point of the double period of seven each, of which hydrogen is the fourteenth member, lies near the point situated at about seventeen units on the negative side of the line of atomic weights, so that five at least of the companions of hydrogen are imaginary, and probably also the sixth.

Hydrogen, therefore, the first-born and king of the elements, sits alone, without brothers or sisters, and seems to resemble the fabled Isis of Egyptian mythology—

> *Ἐγώ εἰμι πᾶν τὸ γεγονός, καὶ ὄν, και ἐσόμενον, καὶ τὸν ἐμὸν πέπλον οὐδείς πω θνητὸς ἀπεκάλυψε.*

## III. The Second and Third Periods; or Carbon-Silicon Double Period.

The following Table gives the atomic weights and valencies of the carbon and silicon periods respectively:—

| *Second Period, or Carbon Period.* | | | *Third Period, or Silicon Period.* | | |
|---|---|---|---|---|---|
| ELEMENT. | ATOMIC WEIGHT. | VALENCY. | ELEMENT. | ATOMIC WEIGHT. | VALENCY. |
| 1. Lithium, .. .. | 7 | Monad. | 1. Sodium, .. .. | 23 | Monad. |
| 2. Beryllium, .. .. | 9 | Dyad. | 2. Magnesium, .. .. | 24 | Dyad. |
| 3. Boron, .. .. | 11 | Triad. | 3. Aluminium, .. .. | 27 | Triad. |
| 4. Carbon, .. .. | 12 | Tetrad. | 4. Silicon, .. .. | 28 | Tetrad. |
| 5. Nitrogen, .. .. | 14 | Triad. | 5. Phosphorus, .. .. | 31 | Triad. |
| 6. Oxygen, . .. | 16 | Dyad. | 6. Sulphur, .. .. | 32 | Dyad. |
| 7. Fluorine, .. .. | 19 | Monad. | 7. Chlorine, .. .. | $35\frac{1}{2}$ | Monad. |

In figure 2 I show, on an enlarged scale, Dr. Reynolds' diagram for the carbon and silicon periods.

This diagram is formed by joining together by right lines the successive fourteen points, commencing with lithium and ending with chlorine. If the points were at random, the number of right lines would be thirteen; and this would be the order of the curve passing through the fourteen points, and an infinite number of such curves could be drawn, and the problem would be geometrically indefinite. The points being supposed placed at random, it is well known that a single general quartic curve can be drawn through them, and one only, thus giving an unique geometrical solution.

A general quartic curve is therefore the simplest solution that the collocation of fourteen points admits of in its most difficult form; but this

solution may become simpler, if the points are arranged by a law or method, and not at random.

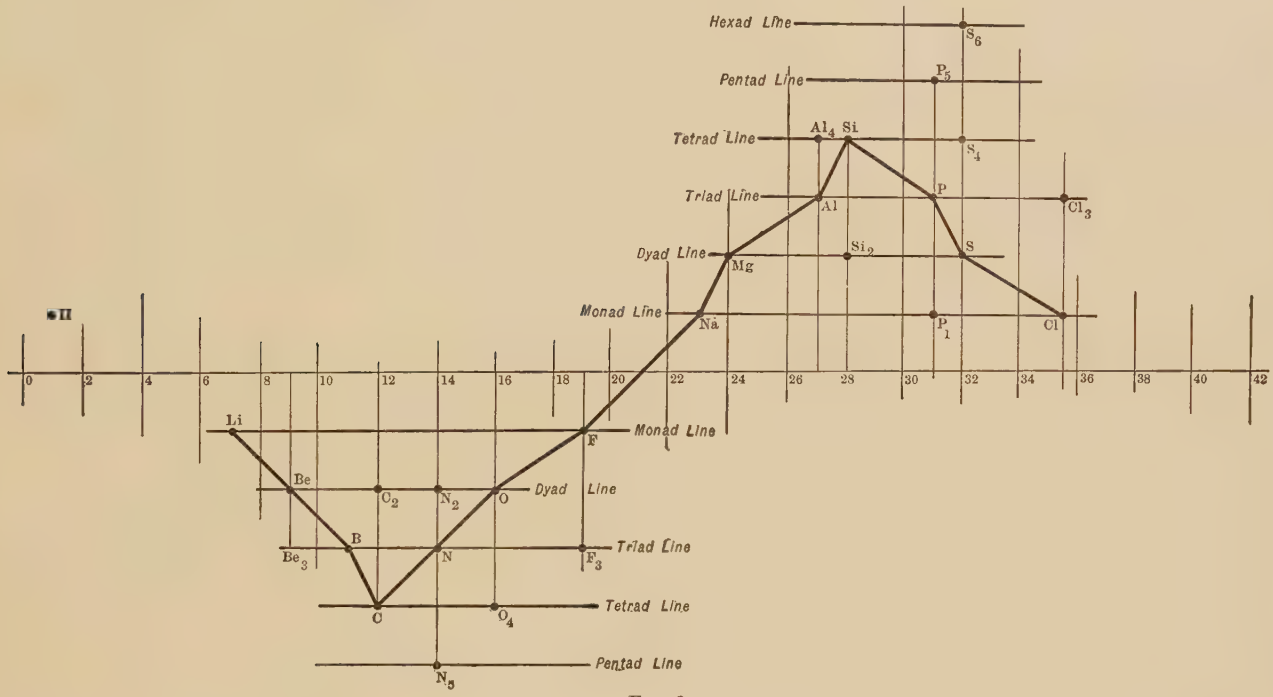



Fig. 2.

In the problem now before us, we find on inspection that five of the fourteen points lie on the same straight line,* viz. :—

C | N | O | Mg | Si.

This is shown in figure 3.

This line joins the two tetrads, carbon and silicon, and passes through the point 20 of the line of atomic weights.

There remain nine points not accounted for, viz. :—

Li | Be | B | F | Na | Al | P | S | Cl.

These nine points are found to lie upon a simple serpentine cubic with one real asymptote, which I have plotted in figure 3.

* The chances are millions to one against this happening at random.

Taking the point 20 for origin, the co-ordinates of the nine points are as follows:—

| ELEMENT. | $x$ | $y$ | ELEMENT. | $x$ | $y$ |
|---|---|---|---|---|---|
| 1. Lithium, .. .. | − 13 | − 1 | 6. Aluminium, .. .. | + 7 | + 3 |
| 2. Beryllium, .. .. | − 11 | − 2 | 7. Phosphorus, .. .. | + 11 | + 3 |
| 3. Boron, .. .. | − 9 | − 3 | 8. Sulphur, .. .. | + 12 | + 2 |
| 4. Fluorine, .. .. | − 1 | − 1 | 9. Chlorine, .. .. | + 15½ | + 1 |
| 5. Sodium, .. .. | + 3 | + 1 | | | |

Writing the equation of the cubic in the form

$$a_1y^3 + a_2y^2x + a_3yx^2 + x^3 + b_1y^2 + b_2yx + b_3x^2 + c_1y + c_2x + d_1 = 0,$$

and grouping together the points that have the same coordinate ($y$), we find

$$y = +1, \qquad \phi_1 + \phi_2x + \phi_3x^2 + x^3 = 0, \qquad \text{Na} \mid \text{cl}$$

where

$$\phi_1 = d_1 + c_1 + b_1 + a_1$$
$$\phi_2 = c_2 + b_2 + a_2$$
$$\phi_3 = b_3 + a_3;$$

or

$$\phi_1 + 3\phi_2 + 9\phi_3 + 27 = 0. \qquad \text{Na}$$

$$\phi_1 + 15{\cdot}5\phi_2 + 240{\cdot}25\phi_3 + 3723{\cdot}875 = 0. \qquad \text{cl}$$

$$y = -1, \qquad f_1 + f_2x + f_3x^2 + x^3 = 0, \qquad \text{F} \mid \text{Li}$$

where

$$f_1 = d_1 - c_1 + b_1 - a_1$$
$$f_2 = c_2 - b_2 + a_2$$
$$f_3 = b_3 - a_3;$$

or

$$f_1 - f_2 + f_3 - 1 = 0. \qquad \text{F}$$

$$f_1 - 13f_2 + 169f_3 - 2197 = 0. \qquad \text{Li}$$

$$y = +3, \qquad F_1 + F_2x + F_3x^2 + x^3 = 0, \qquad \text{Al} \mid \text{P}$$

where
$$F_1 = d_1 + 3c_1 + 9b_1 + 27a_1$$
$$F_2 = c_2 + 3b_2 + 9a_2$$
$$F_3 = b_3 + 3a_3;$$
or
$$F_1 + 7F_2 + 49F_3 + 343 = 0. \qquad \text{Al}$$
$$F_1 + 11F_2 + 121F_3 + 1331 = 0. \qquad \text{P}$$

If we subtract the three foregoing pairs of equations, and reduce to lowest terms, we obtain

(1). $(c_2 + b_2 + a_2) + 18{\cdot}5(b_3 + a_3) + 295{\cdot}75 = 0.$ Na | Cl

(2). $(c_2 - b_2 + a_2) - 14(b_3 - a_3) + 183 = 0.$ F | Li

(3). $(c_2 + 3b_2 + 9a_2) + 18(b_3 + 3a_3) + 247 = 0.$ Al | P

The remaining equations are—

(4). $(d_1 + c_1 + b_1 + a_1) + 3(c_2 + b_2 + a_2) + 9(b_3 + a_3) + 27 = 0.$ Na

(5). $(d_1 - c_1 + b_1 - a_1) - (c_2 - b_2 + a_2) + (b_3 - a_3) - 1 = 0.$ F

(6). $(d_1 + 3c_1 + 9b_1 + 27a_1) + 7(c_2 + 3b_2 + 9a_2) + 49(b_3 + 3a_3) + 343 = 0.$ Al

(7). $(d_1 - 3c_1 + 9b_1 - 27a_1) - 9(c_2 - 3b_2 + 9a_2) + 81(b_3 - 3a_3) - 729 = 0$ B

(8). $(d_1 + 2c_1 + 4b_1 + 8a_1) + 12(c_2 + 2b_2 + 4a_2) + 144(b_3 + 2a_3) + 1728 = 0$ S

(9). $(d_1 - 2c_1 + 4b_1 - 8a_1) - 11(c_2 - 2b_2 + 4a_2) + 121(b_3 - 2a_3) - 1331 = 0.$ Be

The last six equations, by addition and subtraction in pairs, are reduced to

(4)′. $(d_1 + b_1) + (c_2 + 2b_2 + a_2) + (5b_3 + 4a_3) + 13 = 0.$

(5)′. $(c_1 + a_1) + (2c_2 + b_2 + 2a_2) + (4b_3 + 5a_3) + 14 = 0.$

(6)′. $(d_1 + 9b_1) - (c_2 - 24b_2 + 9a_2) + (65b_3 - 48a_3) - 193 = 0.$

(7)′. $(3c_1 + 27a_1) + (8c_2 - 3b_2 + 72a_2) - (16b_3 - 195a_3) + 536 = 0.$

(8)′. $(2d_1 + 8b_1) + (c_2 + 46b_2 + 4a_2) + (265b_3 + 46a_3) + 397 = 0.$

(9)′. $(4c_1 + 16a_1) + (23c_2 + 2b_2 + 92a_2) + (23b_3 + 530a_3) + 3059 = 0.$

From (4)′, (6)′, and (8)′ we find

(4)″. $4d_1 + 5c_2 - 3b_2 + 9a_2 - 10b_3 + 42a_3 + 155 = 0.$

(6)″. $4b_1 - c_2 + 11b_2 - 5a_2 + 30b_3 - 26a_3 - 103 = 0.$

(8)″. $c_2 + 51b_2 + 19a_2 + 420b_3 + 154a_3 + 1051 = 0.$

From (5)′, (7)′, and (9)′ we find

(5)″. $12c_1 + 23c_2 + 15b_2 - 9a_2 + 62b_3 - 30a_3 - 79 = 0.$

(7)″. $12a_1 + c_2 - 3b_2 + 33a_2 - 14b_3 + 90a_3 + 247 = 0.$

(9)″. $14c_2 + b_2 + 51a_2 + 21b_3 + 420a_2 + 2756 = 0.$

From the first three equations we have

(1)′. $2b_2 + 32{\cdot}5b_3 + 4{\cdot}5a_3 + 112{\cdot}75 = 0.$

(2)′. $16a_2 - 66b_3 + 62a_3 - 323 = 0.$

(3)′. $16c_2 + 102b_3 + 198a_3 + 4153 = 0.$

Hence, by elimination, we find

(8)‴. $2694b_3 + 374a_3 + 13601 = 0.$

(9)‴. $2014b_3 + 750a_3 + 1525 = 0.$

$1{,}267264b_3 + 9{,}630400 = 0.$

$b_3 = -\ 7{\cdot}599 \quad a_3 = +\ 18{\cdot}376.$

$c_2 = -\ 438{\cdot}522 \quad b_2 = +\ 25{\cdot}763 \quad a_2 = -\ 82{\cdot}365.$

$d_1 = +\ 502{\cdot}101 \quad c_1 = +\ 838{\cdot}308 \quad b_1 = -\ 81{\cdot}249 \quad a_1 = +\ 102{\cdot}219.$

If $y = mx + n$ be the equation of the asymptotes, then $m$ and $n$ are found from the equations

$$a_1m^3 + a_2m^2 + a_3m + 1 = 0,$$

$$(3a_1m^2 + 2a_2m + a_3)n + (b_1m^2 + b_2m + b_3) = 0,$$

from which we find (for the real asymptote),

$$m = -0{\cdot}045,$$
$$n = -0{\cdot}338.$$

The two remaining asymptotes are imaginary. The complete quartic curve which passes through the fourteen elements is found by multiplying the equation of the cubic by the equation of the lineal branch $x - \lambda y = 0$, where $\lambda = 2$.

Thus,

$$x^4 + A_4x^3y + A_3x^2y^2 + A_2xy^3 + A_1y^4 + B_4x^3 + B_3x^2y + B_2xy^2 + B_1y^3$$
$$+ C_3x^2 + C_2xy + C_1y^2 + D_2x + D_1y + E_1 = 0.$$

$$(x - \lambda y) \times \left\{ \begin{array}{llll} x^3 + a_3x^2y & + a_2xy^2 & + a_1y^3 \\ + b_3x^2 & + b_2xy & + b_1y^2 \\ & + c_2x & + c_1y \\ & & + d_1 \end{array} \right\} = 0.$$

Multiplying and equating coefficients, we find

$$A_4 = a_3 - \lambda, \quad A_3 = a_2 - \lambda a_3, \quad A_2 = a_1 - \lambda a_2, \quad A_1 = -\lambda a_1.$$
$$B_4 = b_3, \quad B_3 = b_2 - \lambda b_3, \quad B_2 = b_1 - \lambda b_2, \quad B_1 = -\lambda b_1.$$
$$C_3 = c_2, \quad C_2 = c_1 - \lambda c_2, \quad C_1 = -\lambda c_1.$$
$$D_2 = d_1, \quad D_1 = -\lambda d_1, \quad E_1 = 0.$$

Substituting the numerical values of $\lambda$ and the cubic coefficients, we find

$$A_4 = +16{\cdot}376, \quad A_3 = -119{\cdot}117, \quad A_2 = +266{\cdot}949, \quad A_1 = -204{\cdot}438.$$
$$B_4 = -7{\cdot}599, \quad B_3 = +40{\cdot}961, \quad B_2 = -132{\cdot}775, \quad B_1 = +162{\cdot}498.$$
$$C_3 = -438{\cdot}522, \quad C_2 = +1715{\cdot}352, \quad C_1 = -1676{\cdot}616.$$
$$D_2 = +502{\cdot}101, \quad D_1 = -1004{\cdot}202. \quad E_1 = 0.$$

This curve is plotted in figure 3, showing it to consist of a lineal branch and a cubic branch. The cubic branch is a simple serpentine cubic, with one real asymptote; and this asymptote and the lineal branch may be regarded as the two real asymptotes of the entire quartic curve. This equa-

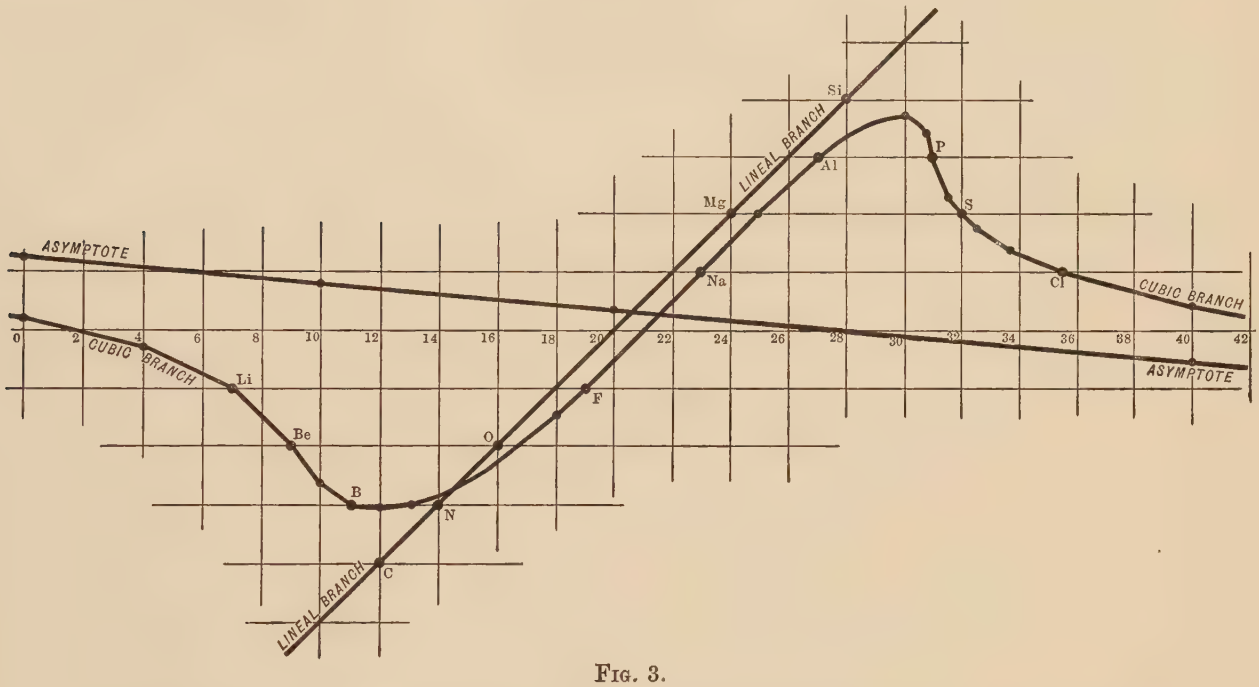



Fig. 3.

tion may be considered as that of a quartic curve, with two real asymptotes, passing accurately through the fourteen points of the second and third chemical periods. Calling the lineal branch the first asymptote, the parameters of the two real asymptotes are—

*Parameters of Quartic Asymptotes.*

| | *m.* | *n.* |
|---|---|---|
| First .. .. | $+\frac{1}{2}$ | 0 |
| Second .. .. | − 0·045 | + 0·338 |

TABLE OF *Fifteen Elements, with Name of Discoverer and Date.*

| ELEMENT. | NAME. | DATE. | ELEMENT. | NAME. | DATE. |
|---|---|---|---|---|---|
| 1. Hydrogen, .. | Cavendish, .. | 1781 | 9. Sodium, .. | Davy, .. | 1807 |
| 2. Lithium, .. | Arfwedson, .. | 1817 | 10. Magnesium, | Davy, .. | 1808 |
| 3. Beryllium, .. | Wohler, .. | 1828 | 11. Aluminium, | Wohler, .. | 1828 |
| 4. Boron, .. | Gay Lussac and Thenard | 1808 | 12. Silicon, .. | Berzelius .. | 1823 |
| 5. Carbon, .. | —— | Prehistoric | 13. Phosphorus, | Brandt, .. | 1669 |
| 6. Nitrogen, .. | Rutherford, .. | 1772 | 14. Sulphur, .. | —— | Prehistoric |
| 7. Oxygen, .. | Priestly, .. | 1774 | 15. Chlorine, .. | Scheele — | 1774 |
| 8. Fluorine, .. | Moissan, .. | 1886 | | | |

## IV. THE FOURTH AND FIFTH PERIODS, OR TITANIUM-GERMANIUM DOUBLE PERIOD.

The following Table gives the atomic weight and valency of each of the elements of the Titanium and Germanium periods.

*Primary Elements.*

| *Fourth Period, or Titanium Period.* | | | *Fifth Period, or Germanium Period.* | | |
|---|---|---|---|---|---|
| ELEMENT. | ATOMIC WEIGHT. | VALENCY. | ELEMENT. | ATOMIC WEIGHT. | VALENCY. |
| 1. Potassium, .. .. | 39 | Monad. | 1. Copper, .. .. | 63 | Monad. |
| 2. Calcium, .. .. | 40 | Dyad. | 2. Zinc, .. .. | 65 | Dyad. |
| 3. Scandium, .. .. | 44 | Triad. | 3. Gallium (*Ekaluminium*). | 69 | Triad. |
| 4. Titanium, .. .. | 48 | Tetrad. | 4. Germanium (*Eka Silicium*). | 72 | Tetrad. |
| 5. Vanadium, .. .. | 51 | Triad. | 5. Arsenic, .. .. | 75 | Triad. |
| 6. Chromium, .. .. | 52 | Dyad. | 6. Selenium, .. .. | 79 | Dyad. |
| 7. Manganese, .. .. | 55 | Monad. | 7. Bromine, .. .. | 80 | Monad. |

*Additional Elements not lying on the Zigzag Line.*

| ELEMENT. | ATOMIC WEIGHT. | VALENCY. |
|---|---|---|
| Iron, .. .. .. | 56 | Dyad. |
| Nickel, .. .. .. | 58·5 | Dyad. |
| Cobalt, .. .. .. | 59 | Dyad. |

At the commencement of the year 1875 there were three unknown elements of this Table, viz. :—

*Gallium*, found in 1875;
*Scandium*, found in 1879;
*Germanium*, found in 1886.

And it is the glory of the Periodic Law of Modern Chemistry that it predicted (approximately) the atomic weights of these three unknown elements and their valencies, which predictions were fulfilled by the successive discovery of the unknown elements.

In figure 4, which is an enlarged representation of the fourth and fifth periods of Dr. Reynolds' curve, we can see that a triad element is wanting between calcium and titanium, and both a triad and tetrad element wanting between zinc and arsenic. These missing elements were predicted and found, as above stated.

It is quite true that there were three well-known elements lying between manganese and copper, viz. iron, nickel, and cobalt; but these three elements did not conform to the rules of the Periodic Law, as interpreted by the zigzag line, and their places have been taken by scandium, gallium, and germanium.

An inspection of figure 4 shows (just as we found in the former curve in the case of the tetrad elements, carbon and silicon) that the line joining the tetrad elements, titanium and germanium, contains five elements, viz.·

Ti | Va | Cu | Ga | Ge,

leaving nine elements free, which sit upon a general cubic—

K | Ca | Sc | Cr | Mn | Zn | As | Se | Br.

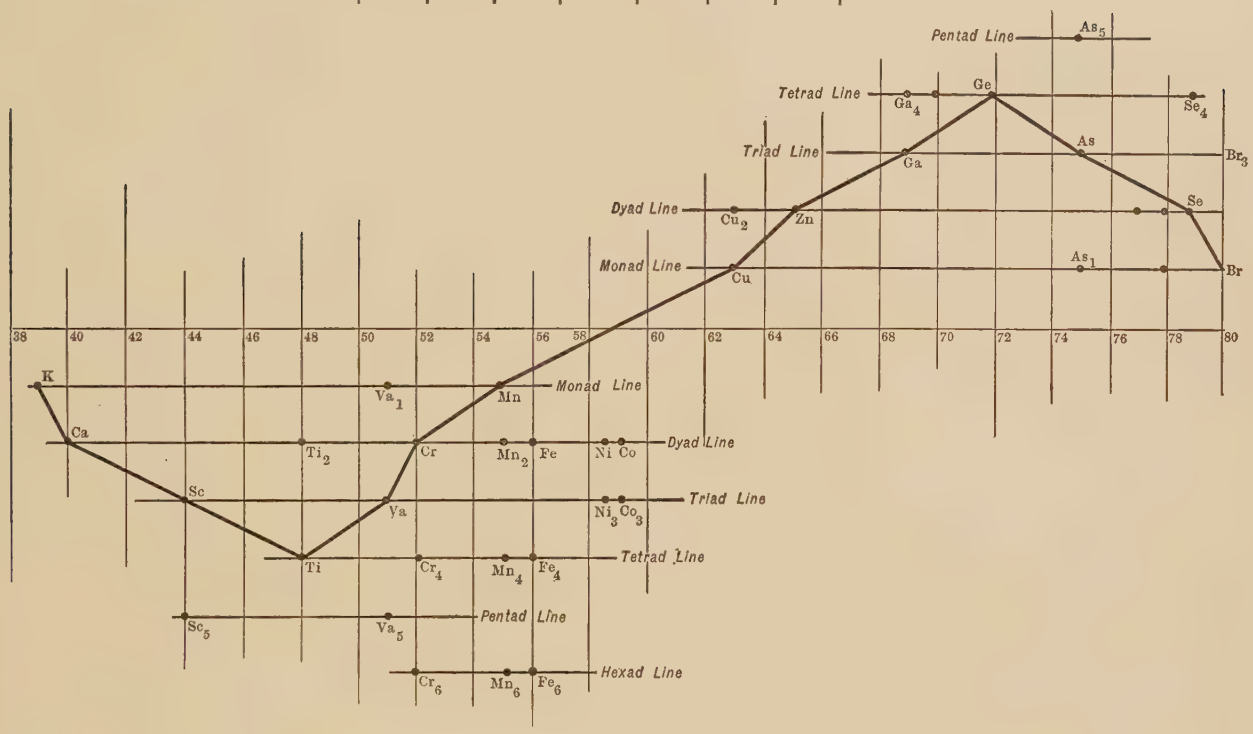



Fig. 4.

The line joining the tetrad elements, titanium and germanium, and which contains five elements, intersects the line of atomic weights at the point 60. Taking this for origin of co-ordinates, we find the following co-ordinates for the nine points of the cubic :—

| ELEMENT. | $x$ | $y$ | ELEMENT. | $x$ | $y$ |
|---|---|---|---|---|---|
| 1. Potassium, .. .. | − 21 | − 1 | 6. Zinc, .. .. | + 5 | + 2 |
| 2. Calcium, .. .. | − 20 | − 2 | 7. Arsenic, .. .. | + 15 | + 3 |
| 3. Scandium, .. .. | − 16 | − 3 | 8. Selenium, .. .. | + 19 | + 2 |
| 4. Chromium, .. .. | − 8 | − 2 | 9. Bromine, .. .. | + 20 | + 1 |
| 5. Manganese, .. .. | − 5 | − 1 | | | |

Grouping together, as before, the points that have the same ordinate $y$, we find—

$$y=-1, \qquad \phi_1 + \phi_2 x + \phi_3 x^2 + x^3 = 0, \qquad \text{Mn} \mid \text{K}$$

where

$$\phi_1 = d_1 - c_1 + b_1 - a_1$$

$$\phi_2 = c_2 - b_2 + a_2$$

$$\phi_3 = b_3 - a_3\,;$$

or

$$\phi_1 - 5\phi_2 + 25\phi_3 - 125 = 0. \qquad \text{Mn}$$

$$\phi_1 - 21\phi_2 + 441\phi_3 - 9261 = 0. \qquad \text{K}$$

$$y=-2, \qquad f_1 + f_2 x + f_3 x^2 + x^3 = 0, \qquad \text{Cr} \mid \text{Ca}$$

where

$$f_1 = d_1 - 2c_1 + 4b_1 - 8a_1$$

$$f_2 = c_2 - 2b_2 + 4a_2$$

$$f_3 = b_3 - 2a_3\,;$$

or

$$f_1 - 8f_2 + 64f_3 - 512 = 0 \qquad \text{Cr}$$

$$f_1 - 20f_2 + 400f_3 - 8000 = 0. \qquad \text{Ca}$$

$$y=+2, \qquad F_1 + F_2 x + F_3 x^2 + x^3 = 0, \qquad \text{Zn} \mid \text{Se}$$

where

$$F_1 = d_1 + 2c_1 + 4b_1 + 8a_1$$

$$F_2 = c_2 + 2b_2 + 4a_2$$

$$F_3 = b_3 + 2a_3\,;$$

or

$$F_1 + 5F_2 + 25F_3 + 125 = 0. \qquad \text{Zn}$$

$$F_1 + 19F_2 + 361F_3 + 6859 = 0. \qquad \text{Se}$$

If we subtract the foregoing equations, in pairs, we find—

(1). $(c_2 - b_2 + a_2) - 26\,(b_3 - a_3) + 571 = 0.$ Mn | K

(2). $(c_2 - 2b_2 + 4a_2) - 28\,(b_3 - 2a_3) + 624 = 0.$ Cr | Ca

(3). $(c_2 + 2b_2 + 4a_2) + 24\,(b_3 + 2a_3) + 481 = 0.$ Zn | Se

The remaining equations are—

(4). $(d_1+c_1+b_1+a_1)+20(c_2+b_2+a_2)+400(b_3+a_3)+8000=0.$ Br

(5). $(d_1-c_1+b_1-a_1)-5(c_2-b_2+a_2)+25(b_3-a_3)-125=0.$ Mn

(6). $(d_1+2c_1+4b_1+8a_1)+5(c_2+2b_2+4a_2)+25(b_3+2a_3)+125=0.$ Zn

(7). $(d_1-2c_1+4b_1-8a_1)-8(c_2-2b_2+4a_2)+64(b_3-2a_3)-512=0.$ Cr

(8). $(d_1+3c_1+9b_1+27a_1)+15(c_2+3b_2+9a_2)+225(b_3+3a_3)+3375=0.$ As

(9). $(d_1-3c_1+9b_1-27a_1)-16(c_2-3b_2+9a_2)+256(b_3-3a_3)-4096=0.$ Se

The last six equations, by addition and subtraction, in pairs, are reduced to

(4)′. $2d_1+2b_1+15c_2+25b_2+15a_2+425b_3+375a_3+7875=0.$

(5)′. $2c_1+2a_1+25c_2+15b_2+25a_2+375b_3+425a_3+8125=0.$

(6)′. $2d_1+8b_1-3c_2+26b_2-12a_2+89b_3-78a_3-387=0.$

(7)′. $4c_1+16a_1+13c_2-6b_2+52a_2-39b_3+178a_3+637=0.$

(8)′. $2d_1+18b_1-c_2+93b_2-9a_2+481b_3-93a_3-721=0.$

(9)′. $6c_1+54a_1+31c_2-3b_2+279a_2-31b_3+1443a_3+7471=0.$

From (4)′, (6)′, and (8)′, we find

(4)″. $6d_1+63c_2+74b_2+72a_2+1611b_3+1578a_3+31887=0.$

(6)″. $6b_1-18c_2+b_2-27a_2-336b_3-453a_3-8262=0.$

(8)″. $24c_2+49b_2+36a_2+714b_3+555a_3+10077=0.$

From (5)′, (7)′, and (9)′, we find

(5)″. $12c_1+187c_2+126b_2+148a_2+3039b_3+3222a_3+64363=0.$

(7)″. $12a_1-37c_2-36b_2+2a_2-789b_3-672a_3-15613=0.$

(9)″. $26c_2+24b_2+49a_2+500b_3+714a_3+11387=0.$

From (1), (2), and (3), we find

$$(1)'. \qquad 6c_2 - 100b_3 + 88a_3 + 3177 = 0.$$

$$(2)'. \qquad 4b_2 + 52b_3 - 8a_3 - 143 = 0.$$

$$(3)'. \qquad 12a_2 + 44b_3 + 112a_3 + 69 = 0.$$

And finally, by elimination,

$$(8)'''. \qquad 276b_3 - 28a_3 - 869 = 0.$$

$$(9)'''. \qquad 1060b_3 - 184a_3 - 4329 = 0.$$

$$21104b_3 - 38684 = 0.$$

$$b_3 = +1{\cdot}833 \qquad a_3 = -12{\cdot}968.$$

$$c_2 = -308{\cdot}753 \qquad b_2 = -14{\cdot}015 \qquad a_2 = +108{\cdot}564.$$

$$d_1 = -284{\cdot}086 \qquad c_1 = +1273{\cdot}720.$$

$$b_1 = +65{\cdot}179 \qquad a_1 = -316{\cdot}732.$$

This cubic curve, like the former one, has one real asymptote, which is found as follows: let its equation be $y = mx + n$,

$$m = +0{\cdot}223 \qquad n = +0{\cdot}165.$$

The complete equation of the quartic curve is found, as before, by multiplying the cubic by the equation of the lineal branch $x - \lambda y = 0$, where $\lambda = 3$; hence we find

$$x^4 + A_4x^3y + A_3x^2y^2 + A_2xy^3 + A_1y^4 + B_4x^3 + B_3x^2y + B_2xy^2 + B_1y^3$$
$$+ C_3x^2 + C_2xy + C_1y^2 + D_2x + D_1y + E_1 = 0,$$

where

$$A_4 = -15{\cdot}968, \quad A_3 = +147{\cdot}468, \quad A_2 = -642{\cdot}424, \quad A_1 = +950{\cdot}196.$$

$$B_4 = +1{\cdot}833, \quad B_3 = -19{\cdot}514, \quad B_2 = +107{\cdot}224, \quad B_1 = -195{\cdot}537.$$

$$C_3 = -308{\cdot}753, \quad C_2 = +2199{\cdot}979, \quad C_1 = -3821{\cdot}160.$$

$$D_2 = -284{\cdot}086, \quad D_1 = +852{\cdot}258, \quad E_1 = 0.$$

This curve is plotted in figure 5, showing it to consist of a lineal branch and a cubic branch. The cubic branch is a whiplash serpentine cubic, with one real asymptote, and this asymptote and the lineal branch may be regarded as the two real asymptotes of the entire quartic curve.

The parameters of the quartic asymptotes are—

*Parameters of Quartic Asymptotes.*

| | *m.* | *n.* |
|---|---|---|
| First .. .. | $+\frac{1}{3}$ | 0 |
| Second .. .. | + 0·223 | + 0·165 |

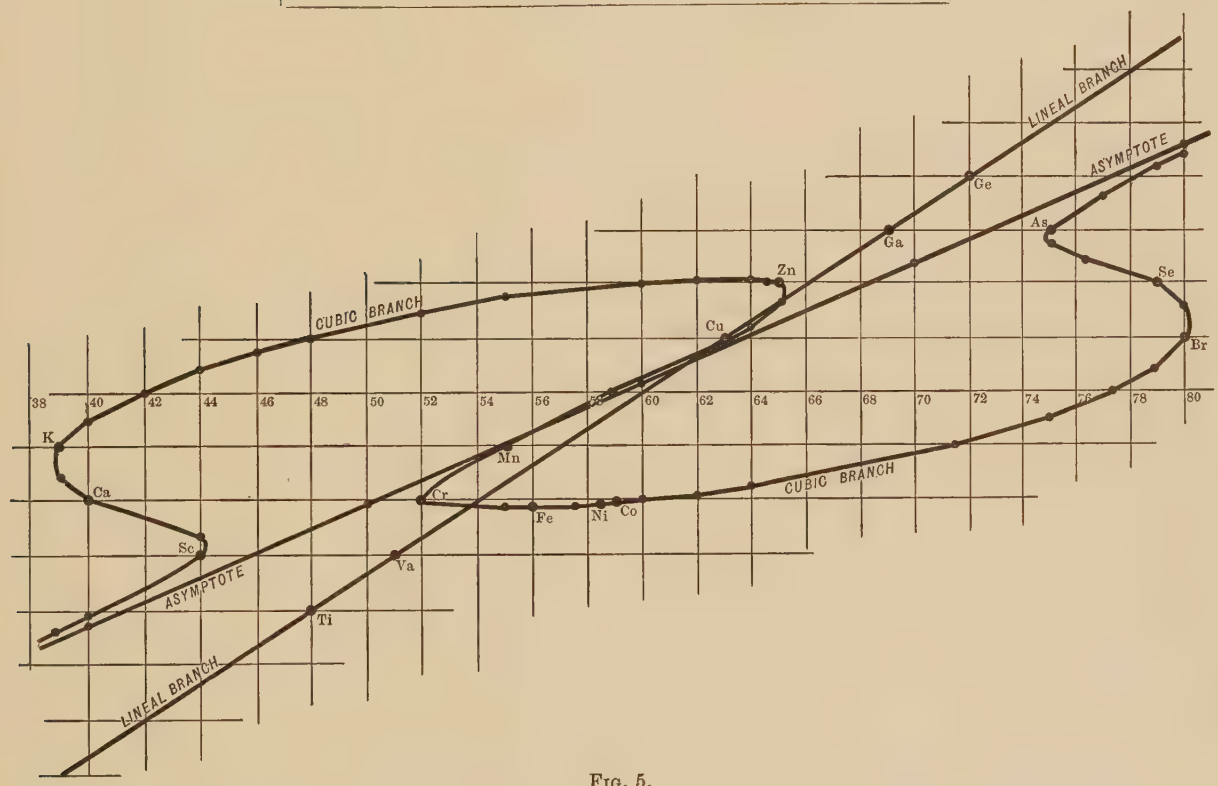



Fig. 5.

This curve was constructed without any reference to the elements iron,

nickel, and cobalt, which appear to evade the zigzag line of the periodic law; but as the cubic curve keeps very close to the paramagnetic dyad line, from the atomic weight 52 (chromium) to the atomic weight 60, a natural place is found upon the curve for the three aberrant elements, as dyads, viz., iron = 56, nickel = 58·5, cobalt = 59, near to their comrade chromium, and they cease to be exceptions, and become, on the contrary, excellent examples of the Periodic Law as interpreted by geometry.

Table *of Fourteen Elements, with Name of Discoverer and Date.**

| ELEMENT. | NAME. | DATE. | ELEMENT. | NAME. | DATE. |
|---|---|---|---|---|---|
| 16. Potassium, . | Davy, . . | 1807 | 23. Copper, . . | — | Prehistoric |
| 17. Calcium, . | Davy, . . | 1808 | 24. Zinc, . . . | Paracelsus, . | xvi. cent. |
| 18. Scandium, . | Nilson, . . | 1879 | 25. Gallium, . | De Boisbaudran | 1875 |
| 19. Titanium, . | Gregor, . . | 1789 | 26. Germanium, | Winkler, . | 1886 |
| 20. Vanadium, . | Sefstrom, . | 1831 | 27. Arsenic, . . | Brandt, . . | 1733 |
| 21. Chromium, . | Vauquelin, . | 1797 | 28. Selenium, . | Berzelius, . | 1817 |
| 22. Manganese, . | Gahn, . . | 1774 | 29. Bromine, . | Balard, . . | 1826 |

*Additional Elements not lying on the Zigzag Line.*

| ELEMENT. | NAME. | DATE. |
|---|---|---|
| 30. Iron, . . | — | Prehistoric |
| 31. Nickel, . | Cronstedt, . | 1751 |
| 32. Cobalt, . | Brandt, . . | 1735 |

* *Vide* Note B., p. 243.

V. Sixth and Seventh Periods, or Zirconium-Tin Double Period.

The following Table gives the atomic weight and valency of the elements of the Zirconium and Tin Periods:—

*Primary Elements.*

| *Sixth Period, or Zirconium Period.* | | | *Seventh Period, or Tin Period.* | | |
|---|---|---|---|---|---|
| ELEMENT. | ATOMIC WEIGHT. | VALENCY. | ELEMENT. | ATOMIC WEIGHT. | VALENCY. |
| 1. Rubidium, .. .. | 85 | Monad. | 1. Silver, .. .. | 108 | Monad. |
| 2. Strontium, .. .. | 87 | Dyad. | 2. Cadmium, .. .. | 112 | Dyad. |
| 3. Yttrium, .. .. | 89 | Triad. | 3. Indium, .. .. | 113 | Triad. |
| 4. Zirconium, .. .. | 90 | Tetrad. | 4. Tin, .. .. | 118 | Tetrad. |
| 5. Niobium, .. .. | 94 | Triad. | 5. Antimony, .. .. | 121 | Triad. |
| 6. Molybdenum, .. | 96 | Dyad. | 6. Tellurium, .. .. | 125 | Dyad. |
| 7. ————, .. | [100] | Monad. | 7. Iodine, .. .. | 127 | Monad. |

*Additional Elements not lying on the Zigzag Line.*

| ELEMENT. | ATOMIC WEIGHT. | VALENCY. |
|---|---|---|
| Rhodium, .. .. | 104 | Triad. |
| Ruthenium, .. .. | 104·5 | Triad. |
| Palladium, .. .. | 106 | Triad. |

In figure 6 I have plotted the thirteen known elements contained in the Zirconium and Tin Double Period, the fourteenth element being unknown as to its atomic weight, but it is a monad and paramagnetic; an

approximation to its atomic weight must be made before we can construct the curve passing through the fourteen points.

In the two former double curves the line joining the two tetrad elements passed through five elements, thus reducing the quartic curve to a straight line and cubic curve; but this cannot be the case with the zirconium-tin curve, in which this line passes through only two known elements, and even if the unknown element lay on this line, the curve would still remain a general quartic. The point at which the line joining zirconium and tin intersects the line of atomic weights is 104.

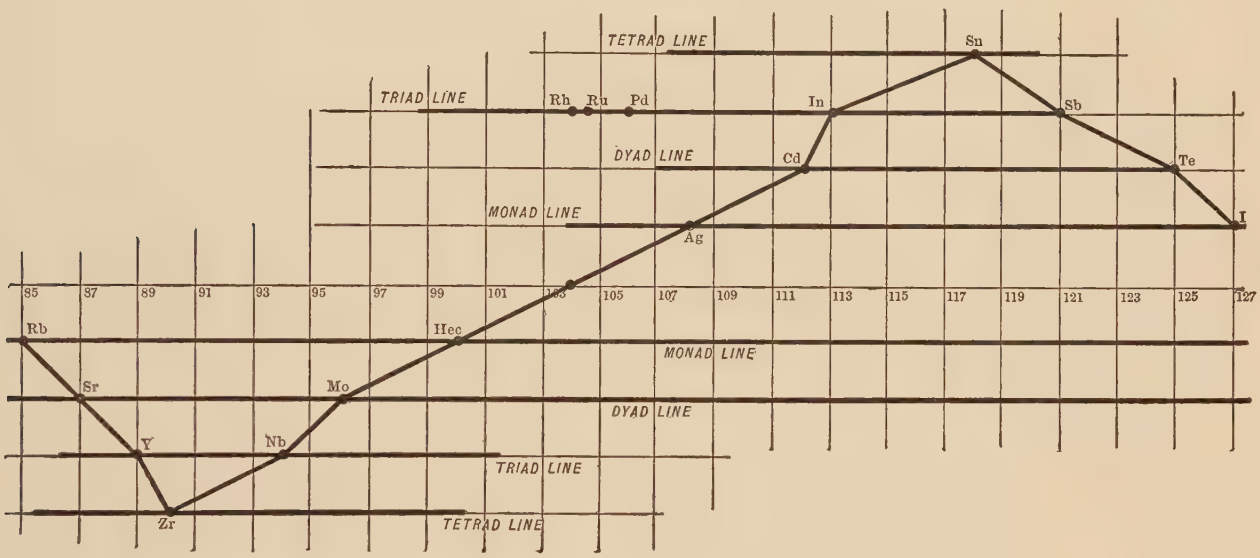



Fig. 6.

We find, on inspection, that cadmium, silver, and molybdenum lie upon one right line, which intersects the line of atomic weights at the same point 104. If the unknown element lie on this line, its atomic weight would be 100. I propose to adopt this atomic weight (provisionally) for the unknown element, which I shall call Hecaton, and content myself for the present with the remark, that whatever its actual atomic weight may prove to be, it is impossible for the zirconium-tin curve to be reduced to a straight line and cubic curve, or to be anything but a general quartic curve.

Taking the point 104 as origin of co-ordinate, we have—

| ELEMENT. | $x$. | $y$. | ELEMENT. | $x$. | $y$. |
|---|---|---|---|---|---|
| 1. Rubidium, .. .. | − 19 | − 1 | 8. Silver, .. .. | + 4 | + 1 |
| 2. Strontium, .. .. | − 17 | − 2 | 9. Cadmium, .. .. | + 8 | + 2 |
| 3. Yttrium, .. .. | − 15 | − 3 | 10. Indium, .. .. | + 9 | + 3 |
| 4. Zirconium, .. .. | − 14 | − 4 | 11. Tin, .. .. | + 14 | + 4 |
| 5. Niobium, .. .. | − 10 | − 3 | 12. Antimony, .. .. | + 17 | + 3 |
| 6. Molybdenum .. | − 8 | − 2 | 13. Tellurium, .. .. | + 21 | + 2 |
| 7. Hecaton, .. .. | − 4 | − 1 | 14. Iodine, .. .. | + 23 | + 1 |

Taking the following as the general equation of the quartic curve,

$$A_1y^4 + A_2y^3x + A_3y^2x^2 + A_4yx^3 + x^4 + B_1y^3 + B_2y^2x + B_3yx^2 + B_4x^3$$
$$+ C_1y^2 + C_2yx + C_3x^2 + D_1y + D_2x + E_1 = 0.$$

Since molybdenum, hecaton, silver, and cadmium are on the right line passing through the origin $x = 4y$ we have, at sight,

(1). $$B_1 + 4B_2 + 16B_3 + 64B_4 = 0.$$

(2). $$D_1 + 4D_2 = 0.$$

(3). $$C_1 + 4C_2 + 16C_3 + 5\phi = 0.$$

(4). $$E_1 - 4\phi = 0.$$

$$[\phi = A_1 + 4A_2 + 16A_3 + 64A_4 + 256.]$$

(5). $$14715 + 637(B_4 + A_4) + 27(C_3 + B_3 + A_3)$$
$$+ (D_2 + C_2 + B_2 + A_2) = 0.$$ I | Ag

(6). $$8671 - 453(B_4 - A_4) + 23(C_3 - B_3 + A_3)$$
$$- (D_2 - C_2 + B_2 - A_2) = 0.$$ Rb | Hec

(7). $$14645 + 673(B_4 + 2A_4) + 29(C_3 + 2B_3 + 4A_3) + (D_2 + 2C_2 + 4B_2 + 8A_2) = 0.$$ Te | Cd

(8). $$8825 - 489(B_4 - 2A_4) + 25(C_3 - 2B_3 + 4A_3) - (D_2 - 2C_2 + 4B_2 - 8A_2) = 0.$$ Sr | Mo

(9). $$9620 + 523(B_4 + 3A_4) + 26(C_3 + 3B_3 + 9A_3) + (D_2 + 3C_2 + 9B_2 + 27A_2) = 0.$$ Sb | In

(10). $$8125 - 475(B_4 - 3A_4) + 25(C_3 - 3B_3 + 9A_3) - (D_2 - 3C_2 + 9B_2 - 27A_2) = 0.$$ Y | Nb

(11). $$6561 + 729(B_4 + 3A_4) + 81(C_3 + 3B_3 + 9A_3) + 9(D_2 + 3C_2 + 9B_2 + 27A_2) + (E_1 + 3D_1 + 9C_1 + 27B_1 + 81A_1) = 0.$$ In

(12). $$10000 - 1000(B_4 - 3A_4) + 100(C_3 - 3B_3 + 9A_3) - 10(D_2 - 3C_2 + 9B_2 - 27A_2) + (E_1 - 3D_1 + 9C_1 - 27B_1 + 81A_1) = 0).$$ Nb

(13). $$38416 + 2744(B_4 + 4A_4) + 196(C_3 + 4B_3 + 16A_3) + 14(D_2 + 4C_2 + 16B_2 + 64A_2) + (E_1 + 4D_1 + 16C_1 + 64B_1 + 256A_1) = 0.$$ Zr

(14). $$38416 - 2744(B_4 - 4A_4) + 196(C_3 - 4B_3 + 16A_3) - 14(D_2 - 4C_2 + 16B_2 - 64A_2) + (E_1 - 4D_1 + 16C_1 - 64B_1 + 256A_1) = 0.$$ Sn

These are the fourteen fundamental equations from which the fourteen coefficients of the quartic are to be calculated. Equations **5 | 6 | 7 | 8 | 9 | 10** give respectively (by addition and subtraction)—

(5)′. $$11693 + 92B_4 + 545A_4 + 25C_3 + 2B_3 + 25A_3 + C_2 + A_2 = 0.$$

(6)′. $$3022 + 545B_4 + 92A_4 + 2C_3 + 25B_3 + 2A_3 + D_2 + B_2 = 0.$$

(7)′. $11735 + 92B_4 + 1162A_4 + 27C_3 + 4B_3 + 108A_3 + 2C_2 + 8A_2 = 0.$

(8)′. $2910 + 581B_4 + 184A_4 + 2C_3 + 54B_3 + 8A_3 + D_2 + 4B_2 = 0.$

(9)′. $5915 + 16B_4 + 998A_4 + 17C_3 + B_3 + 153A_3 + 2C_2 + 18A_2 = 0.$

(10)′. $1495 + 998B_4 + 144A_4 + C_3 + 153B_3 + 9A_3 + 2D_2 + 18B_2 = 0.$

Equations 11 | 12 | 13 | 14 become, after some reductions,

(11)′. $4431 + 271B_4 + 61A_4 + 107C_3 + 57B_3 - 317C_3$
$+ D_2 + 15C_2 + 9B_2 - 185A_2 - 80A_1 = 0.$

(12)′. $3439 + 1727B_4 + 813A_4 + 19C_3 + 321B_3 + 171A_3$
$+ 5D_2 + 3C_2 + 45B_2 + 27A_2 = 0.$

(13)′. $4740 + 1528A_4 - 15C_3 + 480A_3 - 2C_2 + 148A_2 + 45A_1 = 0.$

(14)′. $D_2 + 16B_2 + 120B_3 + 676B_4 = 0.$

Eliminating all the unknowns except $B_4$, $A_4$, $C_3$, $B_3$, $A_3$, we obtain—

(*a*). $2462 + 49B_4 + 552A_4 + 2C_3 + 50B_3 + 32A_3 = 0.$

(*b*). $40795 + 232B_4 - 852A_4 + 85C_3 - 9B_3 - 155A_3 = 0.$

(*c*). $11400 + 1459B_4 + 424A_4 + 8C_3 + 60B_3 + 2A_3 = 0.$

(*d*) $6476\frac{1}{2} - 1283\frac{11}{12}B_4 - 1646\frac{2}{3}A_4 + 30\frac{1}{6}C_3 - 92\frac{1}{2}B_3 - 159\frac{1}{3}A_3 = 0.$

(*e*). $61162\frac{1}{2} - 3422\frac{1}{4}B_4 - 39304A_4 + 437\frac{1}{2}C_3 - 426\frac{1}{2}B_3 - 3590A_3 = 0.$

Solving these five equations, we find

$$B_4 = -4{\cdot}031088, \quad A_4 = -10{\cdot}775608,$$

$$C_3 = -421{\cdot}557359, \quad B_3 = +37{\cdot}248605, \quad A_3 = +83{\cdot}048153.$$

The remaining coefficients are easily calcalated, and are as follows:—

$D_2 = +24{\cdot}420$, $C_2 = +3348{\cdot}429$, $B_2 = -110{\cdot}584$, $A_2 = -409{\cdot}429$.

$E_1 = -51{\cdot}868$, $D_1 = -97{\cdot}680$, $C_1 = -6583{\cdot}969$.

$B_1 = +104{\cdot}336$, $A_1 = +729{\cdot}645$, $\phi = -12{\cdot}967$.

This represents a general quartic curve, with two real asymptotes, found by the following equations:—

$$y = mx + n,$$

$$A_1m^4 + A_2m^3 + A_3m^2 + A_4m + 1 = 0,$$

$$\{4m^3A_1 + 3m^2A_2 + 2mA_3 + A_4\}\, n + \{m^3B_1 + m^2B_2 + mB_3 + B_4\} = 0.$$

*Parameters of Quartic Asymptotes.*

| | $m$ | $n$ |
|---|---|---|
| First, .. .. | + 0·27762 | − 0·00635 |
| Second, .. .. | + 0·22303 | − 0·0393 |

This curve is plotted in figure 7, and consists of two real ovals

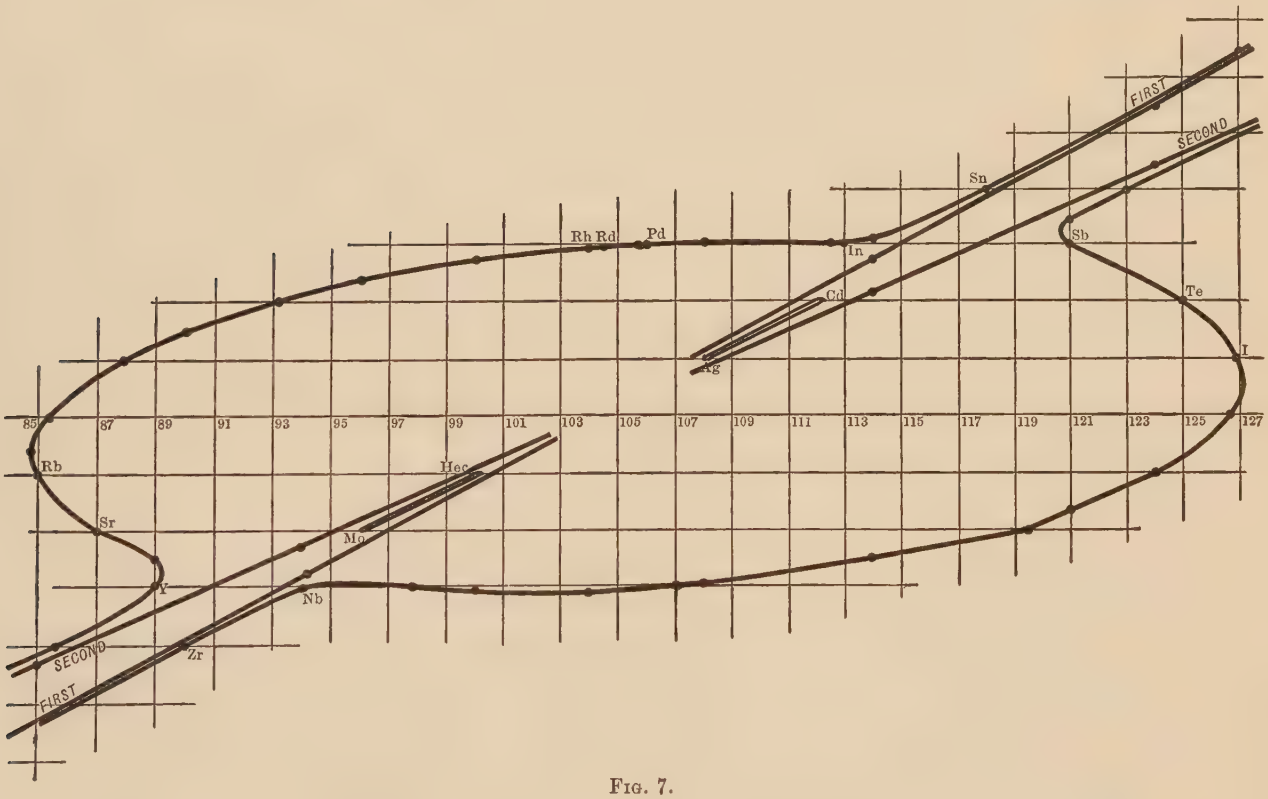



Fig. 7.

lying inside the acute angles between the asymptotes, and two infinite

branches occupying the obtuse angles of the asymptotes. One of the ovals contains silver and cadmium, and the other contains molybdenum, and the as yet undiscovered hecaton. Rhodium, ruthenium, and palladium find their natural places on or near the triad line, passing through indium, although they were not taken into consideration at all in the calculation of the curve. The curve, in fact, nearly coincides with the horizontal triad line, from the atomic weight 104 to 113.

TABLE OF *Elements, with Name of Discoverer and Date.*

| ELEMENT. | NAME. | DATE. | ELEMENT. | NAME. | DATE. |
|---|---|---|---|---|---|
| 33. Rubidium, . | Kirchoff and Bunsen, | 1860 | 39. Silver, . | Prehistoric, . | — |
| 34. Strontium, . | Davy, . . | 1808 | 40. Cadmium, . | Stromeyer, . | 1818 |
| 35. Yttrium, . | Wohler, . | 1828 | 41. Indium, . | Reich & Richter | 1875 |
| 36. Zirconium, . | Berzelius, . | 1824 | 42. Tin, . . | Prehistoric, . | — |
| 37. Niobium, . | Hatchett, . | 1801 | 43. Antimony, . | Basil Valentine | 1490-1500 |
| 38. Molybdenum, | Hjelm, . . | 1782 | 44. Tellurium, . | Klaproth, . | 1798 |
| Hecaton, . | —— | — | 45. Iodine, . | Courtois, . | 1812 |

*Additional Elements not lying on the Zigzag Line.*

| ELEMENT | NAME. | DATE. |
|---|---|---|
| 46. Rhodium, . | Wollaston, . | 1804 |
| 47. Ruthenium . | Claus, . . | 1846 |
| 48. Palladium, . | Wollaston, . | 1803 |

## VI. FORECAST OF ATOMIC WEIGHTS AND VALENCIES OF CHEMICAL ELEMENTS BEYOND IODINE.

In the foregoing discussion, I have included all the known elements from hydrogen to iodine (both included), forty-eight in all, and also the

hypothetical element, hecaton, supposed to have an atomic weight = 100 q. p., but not yet discovered as a terrestrial element, although predicted by the Periodic Law.

The six chemical periods (of seven each) taken in pairs, have given me three curves, which have this in common, that they are of the quartic degree, with two real asymptotes, the parameters of which are—

*Parameters of Quartic Curves (First Asymptote),*

$$y = mx + n.$$

| NO. | DOUBLE PERIOD. | $m$ | $n$ | ORIGIN OF CO-ORDINATES. |
|---|---|---|---|---|
| 1°. | Carbon—Silicon, .. .. | + 0·500′ | 0·000 | 20·0 |
| 2°. | Titanium—Germanium, .. | + 0·333′ | 0·000 | 60·0 |
| 3°. | Zirconium—Tin, .. .. | + 0·27762 | − 0·00635 | 104·0 |

*Parameters of Quartic Curves (Second Asymptote),*

$$y = m'x + n'.$$

| NO. | DOUBLE PERIOD. | $m'$ | $n'$ | ORIGIN OF CO-ORDINATES. |
|---|---|---|---|---|
| 1°. | Carbon—Silicon, .. .. | − 0·045 | + 0·338 | 20 |
| 2°. | Titanium—Germanium, .. | + 0·223 | + 0·165 | 60 |
| 3°. | Zirconium—Tin, .. .. | + 0·22303 | − 0·0393 | 104 |

If we plot (by Cartesian co-ordinates) $m$, $m'$, $n$, $n'$ with reference to the atomic weights that denote the origin of co-ordinates of the successive curves, we find that $m$ and $n'$ may be well represented by rectangular hyperbolas, and that $m'$ and $n$ may be calculated by linear interpolation.*

* *Vide* Note C, p. 245.

## VII. The Eighth and Ninth Periods, or Cerium — Double Period.

There are only seven known elements, belonging to this double period, out of the fourteen that lie on the zigzag line, and these seven elements are not situated in positions favourable for calculating the properties of the remainder, as I shall now show.

### Primary Elements.

| *Eighth Period, or Cerium Period.* | | | *Ninth Period.* | | |
|---|---|---|---|---|---|
| ELEMENT. | ATOMIC WEIGHT. | VALENCY. | ELEMENT. | ATOMIC WEIGHT. | VALENCY. |
| 1. Caesium, . | 133 | Monad. | 1. ——— . | [156·21] | Monad. |
| 2. Barium, . | 137 | Dyad. | 2. ——— . | [160·44] | Dyad. |
| 3. Lanthanum, . | 138 | Triad. | 3. ——— . | — | Triad. |
| 4. Cerium, . | 140 | Tetrad. | 4. ——— . | — | Tetrad. |
| 5. Didymium, . | 142 | Triad. | 5. Erbium, . | 166 | Triad. |
| 6. ——— . | [143·51] | Dyad. | 6. ——— . | — | Dyad. |
| 7. ——— . | [147·74] | Monad. | 7. Ytterbium, . | 173 | Monad. |

*Additional Element not lying on the Zigzag Line.*

| ELEMENT. | ATOMIC WEIGHT. | VALENCY. |
|---|---|---|
| Samarium, . | 150 | Triad. |

It would be easy to calculate the asymptotes of this curve from the data given in Note C, provided we knew what point on the line of atomic weights to take for the origin of co-ordinates. In the three curves already constructed, this point was taken where the line joining the tetrad elements intersects the line of atomic weights indicating the points 20, 60, and 104, respectively. In the present case we have only one tetrad element

given, Cerium, and the solitary clue we have to the exact place of the asymptotes is to follow the type curve (figure 7), and to place them so as to keep the elements Erbium, Didymium, and Cerium, to the right hand of the asymptotes, and the elements Lanthanum, Barium, and Cæsium, to the left hand; and this problem is rendered more difficult by the fact that $m$, $n$, and $m'$, $n'$ are functions of $x$, which determines the position of the origin of co-ordinates. I find that $x = 151{\cdot}8$ best fulfils these conditions and others, and have, accordingly, constructed figure 8 to represent the double

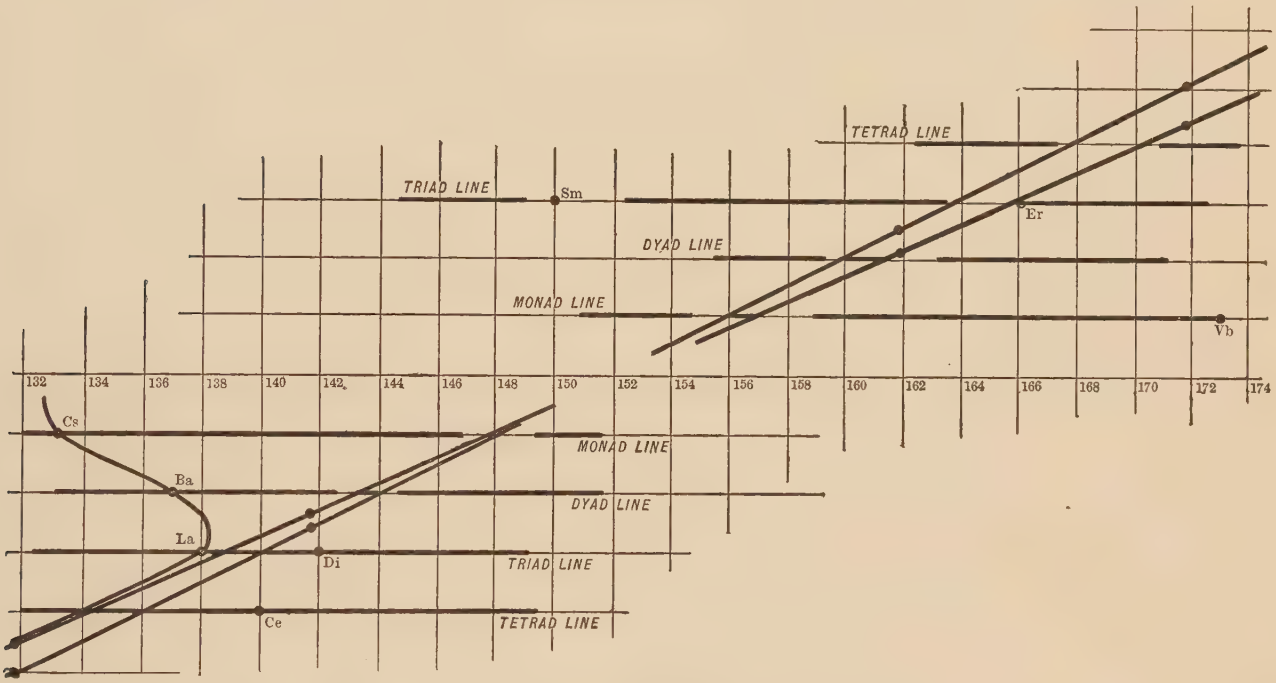



Fig. 8.

period Cerium —. The parameters of the asymptotes are calculated from Note C, by using 151·8 as the value of $x$.

*Parameters of Quartic Asymptotes.*

| | $m$ | $n$ | ORIGIN. |
|---|---|---|---|
| First, .. .. | + 0·25088 | − 0·013148 | 151·8 |
| Second, .. .. | + 0·223033 | − 0·065977 | 151·8 |

The two pairs of elements (Dyad and Monad) that lie on the oval branches inside the asymptotes are as yet unknown, but their positions are marked out by the horizontal lines drawn between the asymptotes, on the monad and dyad lines, and their atomic weights may be forecast within narrow limits. I have calculated these limits as follows :—

*Forecasts of Atomic Weights.*

| NAME OF ELEMENT. | LIMITS OF ATOMIC WEIGHT. |
|---|---|
| 1. Sixth of *Cerium* period, .. | 143·13 to 143·89 |
| 2. Seventh of *Cerium* period, .. | 147·61 to 147·87 |
| 3. First of 9th period, .. .. | 155·84 to 156·58 |
| 4. Second of 9th period, .. | 159·82 to 161·06 |

TABLE OF ELEMENTS, *with Name of Discoverer and Date.*

| ELEMENT. | NAME. | DATE. |
|---|---|---|
| 49. Cæsium, .. .. | Bunsen and Kirkchoff, .. | 1860 |
| 50. Barium, .. .. | Davy, .. .. .. | 1808 |
| 51. Lanthanum, .. | Mosander, .. .. | 1839 |
| 52. Cerium, .. .. | Klaproth, .. .. | 1803 |
| 53. Didymium,.. .. | Mosander, .. .. | 1841 |
| 54. Erbium, .. .. | Mosander, .. .. | 1843 |
| 55. Ytterbium,.. .. | Marignac, .. .. | 1878 |
| *Additional Elements not lying on Zigzag Line.* | | |
| 56. Samarium,.. .. | De Boisbaudran, .. | 1878 |

## VIII. The Tenth and Eleventh Periods, or — Lead Double Period.

There are seven elements known, out of the fourteen which comprise the full number of the elements of this double period lying on the zigzag line, and they are more favourably situated for determining the remaining elements, than was the case in section vii.

*Elements lying on the Zigzag Line.*

| *Tenth Period.* | | | *Eleventh Period, or Lead Period.* | | |
|---|---|---|---|---|---|
| ELEMENT. | ATOMIC WEIGHT. | VALENCY. | ELEMENT. | ATOMIC WEIGHT. | VALENCY. |
| 1. ——— . | — | Monad. | 1. Gold, . . | 196·7 | Monad. |
| 2. ——— . | — | Dyad. | 2. Mercury, . | 200·25 | Dyad. |
| 3. ——— . | — | Triad. | 3. Thallium, . | 204 | Triad. |
| 4. ——— . | — | Tetrad. | 4. Lead, . . | 207 | Tetrad. |
| 5. Tantalum, . | 182 | Triad. | 5. Bismuth, . | 208 | Triad. |
| 6. Wolfram, . | 183·6 | Dyad. | 6. ——— . | — | Dyad. |
| 7. ——— . | [188] | Monad. | 7. ——— . | — | Monad. |

*Elements not lying on the Zigzag Line.*

| ELEMENT. | ATOMIC WEIGHT. | VALENCY. |
|---|---|---|
| Osmium, . . | 191·12 | Triad. |
| Iridium, . . | 192·50 | Triad. |
| Platinum, . . | 194·30 | Triad. |

In the case of this curve we have a valuable clue to guide us in calculating the position of the two real asymptotes; for, if we refer to the type curve, figure 7, we see that not only the line joining the tetrad elements passes through the origin, but also the line joining the elements lying on the two real ovals passes through the same point.

Now, in the case of the present curve, although we have only one tetrad, yet we have three known elements (mercury, gold, wolfram) placed on the ovals in one right line passing through the point 192, just as the line joining cadmium, silver, and molybdenum passed through the point 104, and also the line joining the tetrads tin and zirconium passed through the same point.

Taking, therefore, the point 192 as our origin of co-ordinates, I find from the equations of Note C—

*Parameters of Quartic Asymptotes.*

| | *m* | *n* | ORIGIN. |
|---|---|---|---|
| First, .. .. | + 0·23668 | − 0·01905 | 192 |
| Second, .. .. | + 0·2230352 | − 0·129334 | 192 |

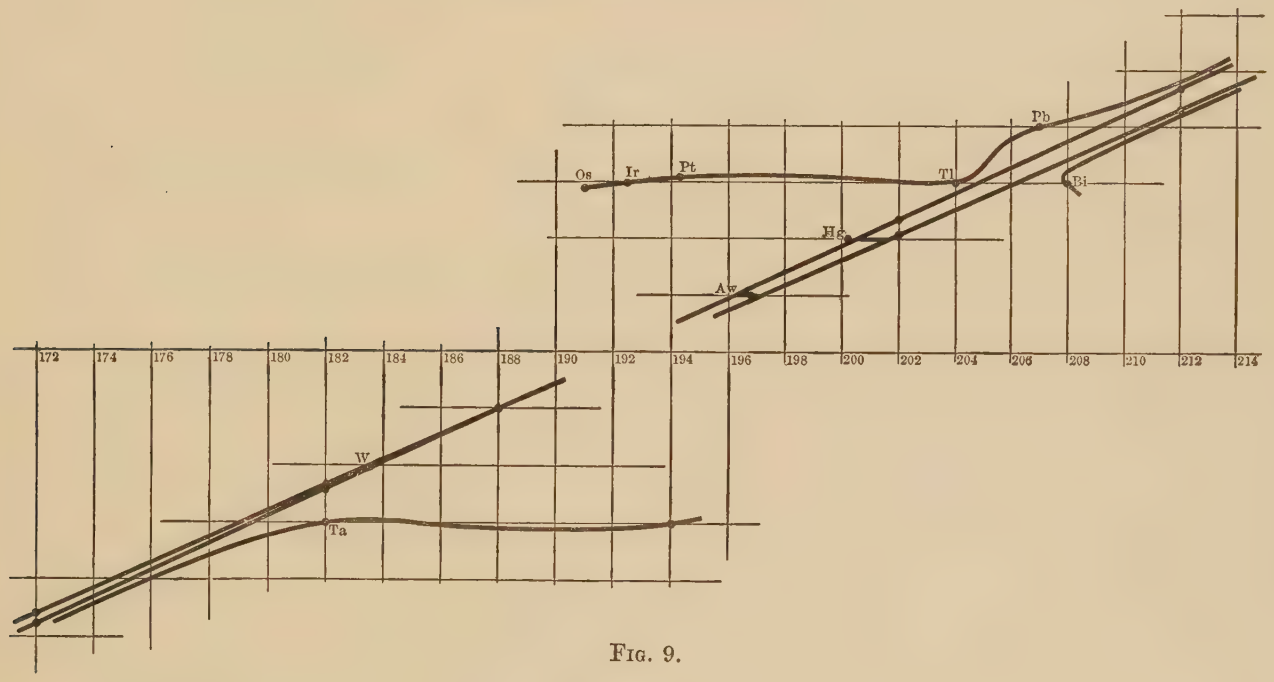



Fig. 9.

Using these data, I have drawn in figure 9 all that I know of this remarkable double period, which is of extreme interest, from the fact that

three of the four elements which should occur on the two real ovals, viz., mercury, gold, wolfram, are long known and well established. Now, as I have already explained, the four elements belonging to the real ovals admit of an accurate forecast of their atomic weights, which, in the present case, can be compared with observation, and a test is thus provided to measure the probability of the laws of variation of parameters of the quartic asymptotes which I have laid down. The following Table contains this comparison :—

*Atomic Weights of Mercury, Gold, and Wolfram, by Calculation and Observation.*

| | ATOMIC WEIGHT CALCULATED. | ATOMIC WEIGHT OBSERVED. | AUTHORITY. |
|---|---|---|---|
| 1. Mercury, . . | 200·5307 to 201·5471 = 201·039 ± 0·508 | 200·25 | Vander Plaatz.<br>Ostwald. |
| 2. Gold, . . | 196·3056 to 197·0635 = 196·864 ± 0·379 | 196·7 | Thorpe.<br>Laurie.<br>Krüss. |
| 3. Wolfram, . | 183·6127 to 183·6303 = 183·621 ± 0·009 | 183·605 | Clarke.<br>Meyer.<br>Seubert. |
| 4. Unknown monad,. | 187·8554 to 188·0963 = 187·976 ± 0·120 | — | — |

The following Table contains the names of the observers who discovered these elements, and the dates of their discovery :—

Table of *Elements lying on the Zigzag Line, with name of Discoverer and Date.*

| ELEMENT. | NAME. | DATE. |
|---|---|---|
| 57. Tantalum, .. .. | Ekeberg, .. .. | 1802. |
| 58. Wolfram, .. .. | D'Elhujar, .. .. | 1783. |
| 59. Gold, .. .. | ——— .. .. | Prehistoric. |
| 60. Mercury, .. .. | ——— .. .. | Almost Prehistoric. |
| 61. Thallium, .. .. | Crookes, .. .. | 1861. |
| 62. Lead, .. .. | ——— .. .. | Prehistoric. |
| 63. Bismuth, .. .. | Basil Valentine, .. .. | xv. Century. |
| *Elements not lying on the Zigzag Line.* | | |
| 64. Osmium, .. .. | Tennant, .. .. | 1804 |
| 65. Iridium, .. .. | Tennant, .. .. | 1804 |
| 66. Platinum, .. .. | Wood, .. .. | 1748 |

## IX. The Twelfth and Thirteenth Periods, or Thorium — Double Period.

Of the supposed fourteen elements lying on the zigzag line only two are known, viz., thorium, the first tetrad, and uranium, the succeeding triad.

It is not possible from these slender data to form an opinion of the whole double period beyond this, that the elements lie farther apart than in the preceding curves, and that probably the element forming forces were dying out when the more complex ones were made.

*Name of Discoverer and Date.*

| ELEMENT. | NAME. | DATE. |
|---|---|---|
| 67. Thorium, .. .. | Berzelius, — — | 1828 |
| 68. Uranium, .. .. | Klaproth, — — | 1789 |

## NOTE A.

### Fourier Curves.

If we apply Fourier's method to the elements of the two periods of seven, following hydrogen, viz.—

| *Second Period.* | *Third Period.* |
|---|---|
| 1. Lithium, | 1. Sodium, |
| 2. Beryllium, | 2. Magnesium, |
| 3. Boron, | 3. Aluminium, |
| 4. Carbon, | 4. Silicon, |
| 5. Nitrogen, | 5. Phosphorus, |
| 6. Oxygen, | 6. Sulphur, |
| 7. Fluorine, | 7. Chlorine, |

we find, dividing the circle into sixteen equal parts,

$$\phi = n\,(t - t^\circ), \quad \text{where} \quad n = 11^\circ\ 30', \ t_0 = 4.$$

*First Harmonic*—

$$a_0 + a_1 \cos\phi + b_1 \sin\phi, \quad a_0 = -\,0{\cdot}055, \quad a_1 = -\,0{\cdot}0882, \quad b_1 = +\,3{\cdot}147.$$

*Second Harmonic*—

$$a_2 \cos 2\phi + b_2 \sin 2\phi, \quad a_2 = +\,0{\cdot}0106, \quad b_2 = -\,0{\cdot}1088.$$

*Third Harmonic*—

$$a_3 \cos 3\phi + b_3 \sin 3\phi, \quad a_3 = +\,0{\cdot}074, \quad b_3 = -\,0{\cdot}466.$$

*Fourth Harmonic*—

$$a_4 \cos 4\phi + b_4 \sin 4\phi, \quad a_4 = +\,0{\cdot}050, \quad b_4 = +\,0{\cdot}01875.$$

Calculating the First Harmonic, and comparing it with observation, I find the following Table:—

Table of *Observed and Calculated Valencies.*

| ELEMENT. | OBSERVED. | CALCULATED. | DIFFERENCE. | ELEMENT. | OBSERVED. | CALCULATED. | DIFFERENCE. |
|---|---|---|---|---|---|---|---|
| Lithium, .. | + 1·000 | + 1·622 | − 0·622 | Sodium, .. | − 1·000 | − 1·777 | + 0·777 |
| Beryllium, .. | + 2·000 | + 2·514 | − 0·514 | Magnesium, | − 2·000 | − 2·217 | + 0·217 |
| Boron, .. | + 3·000 | + 3·015 | − 0·015 | Aluminium, | − 3·000 | − 3·142 | + 0·142 |
| Carbon, .. | + 4·000 | + 3·092 | + 0·908 | Silicon, .. | − 4·000 | − 3·202 | − 0·798 |
| Nitrogen, .. | + 3·000 | + 2·887 | + 0·113 | Phosphorus, | − 3·000 | − 2·719 | − 0·281 |
| Oxygen, .. | + 2·000 | + 2·232 | − 0·232 | Sulphur, .. | − 2·000 | − 2·342 | + 0·342 |
| Fluorine, .. | + 1·000 | + 0·646 | + 0·354 | Chlorine, .. | − 1·000 | − 0·451 | − 0·549 |

This Table is a lamentable example of approximation; for no chemist could accept the calculated valency of any of the fourteen elements, with the single exception of Boron. It is not worth while to calculate the higher Harmonics for all the elements; I shall select for calculation the two most refractory, but at the same time the two most important elements, viz., Carbon and Silicon.

TABLE OF *Observed and Calculated Valencies of Carbon and Silicon.*

| | OBSERVED. | CALCULATED. | | | | DIFFERENCE. |
|---|---|---|---|---|---|---|
| | | FIRST HARMONIC. | FIRST TWO HARMONICS. | FIRST THREE HARMONICS. | FIRST FOUR HARMONICS. | |
| Carbon, .. | +4·000 | +3·092 | +3·0814 | +3·5474 | +3·5974 | +0·4026 |
| Silicon, .. | −4·000 | −3·202 | −3·2126 | −3·6786 | −3·6286 | −0·3714 |

The differences between calculation and observation, even after using four Harmonics, are simply intolerable; they amount, in the case of Carbon, to 10·065 per cent.; and in the case of Silicon to 9·285 per cent.

---

## NOTE B.

### THE DOUBLE PERIOD (Titanium—Germanium).

We meet here, for the first time, well-known elements not lying on or near the zigzag line joining the successive elements. When valencies occurred on the zigzag line itself, such as Scandium, Gallium, and Germanium, it was easy to predict their approximate atomic weights, with the aid of the "Rule of Three"; but in the case of Iron, Nickel, and Cobalt, the zigzag line is helpless. If we replace the zigzag line by the geometrical curve passing through the fourteen given points, it turns out to have the shape figured in figure 5; and Iron, Nickel, and Cobalt find their places on the curve near the dyad line, corresponding to $y = -2$. Substituting this value of $y$ in the general cubic equation, we find

1°. $$x^3 + (b_3 - 2a_3)\,x^2 + (c_2 - 2b_2 + 4a_2)\,x + (d - 2c_1 + 4b_1 - 8a_1) = 0,$$

or

$$x^3 + 27{\cdot}769x^2 + 153{\cdot}533x - 36{\cdot}954 = 0.$$

The roots of this equation give the following atomic weights:—

(1) $x = 40$ (*Calcium*).

(2) $x = 52$ (*Chromium*).

(3) $x = 60{\cdot}23$ (crosses the dyad line).

(4) $x = 54$ (lineal branch).

Hence we see that from 52–60 the curve keeps quite close to the dyad line, leaving abundant room for elements, such as Iron, Nickel, Cobalt, closely related to Chromium.

2°. If we draw the monad line ($y = -1$), containing Potassium and Manganese, we find

$$x^3 + (b_3 - a_3)\,x^2 + (c_2 - b_2 + a_2)\,x + (d_1 - c_1 + b_1 - a_1) = 0,$$

or

$$x^3 + 14{\cdot}801x^2 - 186{\cdot}174x - 1175{\cdot}895 = 0.$$

The roots of this equation give the following Atomic Weights:—

(1) $x = 39$ (*Potassium*).

(2) $x = 55$ (Manganese).

(3) $x = 71{\cdot}199$ (crosses the monad line).

(4) $x = 57$ (lineal branch).

3°. If we draw the monad line ($y = +1$) passing through Bromine and Copper, we find

$$x^3 + (b_3 + a_3)\,x^2 + (c_2 + b_2 + a_2)\,x + (d_1 + c_1 + b_1 + a_1) = 0,$$

or

$$x^3 - 11{\cdot}135x^2 - 214{\cdot}204x^2 + 738{\cdot}081 = 0.$$

The roots of this equation give the following atomic weights:—

(1) $x = 80$ (Bromine).

(2) $x = 63{\cdot}09$ (companion to Copper).

(3) $x = 48{\cdot}05$ (crosses the monad line).

(4) $x = 63$ (Copper) (lineal branch).

The points 63 and 63·09, in which the monad line intersects the lineal and cubic branches of the curve, are so close as to suggest that copper may be a compound element, consisting of two whose atomic weights are 63 and 63·09. The recognised values of the atomic weights of copper range from 63 to 63·33.

4°. If we draw the dyad line ($y = +2$), passing through Selenium and Zinc, we find

$$x^3 + (b_3 + 2a_3)\,x^2 + (c_2 + 2b_2 + 4a_2)\,x + (d_1 + 2c_1 + 4b_1 + 8a_1) = 0,$$

or

$$x^3 - 24{\cdot}103x^2 + 97{\cdot}473x - 9{\cdot}786 = 0.$$

The roots of this equation give the following atomic weights :—

(1) $x = 79$ (Selenium).

(2) $x = 65$ (Zinc).

(3) $x = 60{\cdot}1035$ (crosses the Dyad line.

(4) $x = 66$ (lineal branch).

---

## NOTE C.

### Variation of Parameters of the Real Asymptotes.

The following memoranda contain the details of the construction of the rectangular hyperbolas—

1°. *Calculation of m*—

$$(m - m_0)(x - x_0) = k^2.$$

Introducing the values of $m$ and $x$ from the three equations, we find

$$m_0 = +\,0{\cdot}18054.$$

$$x_0 = -\,16{\cdot}671.$$

$$k^2 - m_0x_0 = +\,14{\cdot}7247.$$

2°. *Colculation of n′*—

$$(n_0' + n')(x_0 + x) = k^2.$$

$$n_0' = +\,0{\cdot}6267.$$

$$x_0 = +\,163{\cdot}053.$$

$$k^2 - n_0'x_0 = +\,74{\cdot}4059.$$

3°. *Calculation of m′*—

$$(m_0' - m')(x - x_0) = k^2.$$

$$m_0' = +\,0{\cdot}22305.$$

$$x_0 = +\,19{\cdot}9915.$$

$$k^2 + x_0m_0' = +\,4{\cdot}4614.$$

4°. *Calculation of n*—

By linear interpolation.

The foregoing calculations of the laws of variation of the parameters of the quartic asymptotes are of very different values.

The coefficients $m$ and $m'$ depend only on $A_1 \mid A_2 \mid A_3 \mid A_4$, and those of $n$ and $n'$ depend not only on $A_1 \mid A_2 \mid A_3 \mid A_4$, but also upon $B_1 \mid B_2 \mid B_3 \mid B_4$; accordingly, *cæteris paribus*, the first two are of more importance, but the law of variation of $m$ is more clearly marked than that of $m'$. I have plotted and shown in figure 10 the law of variation of $m$.

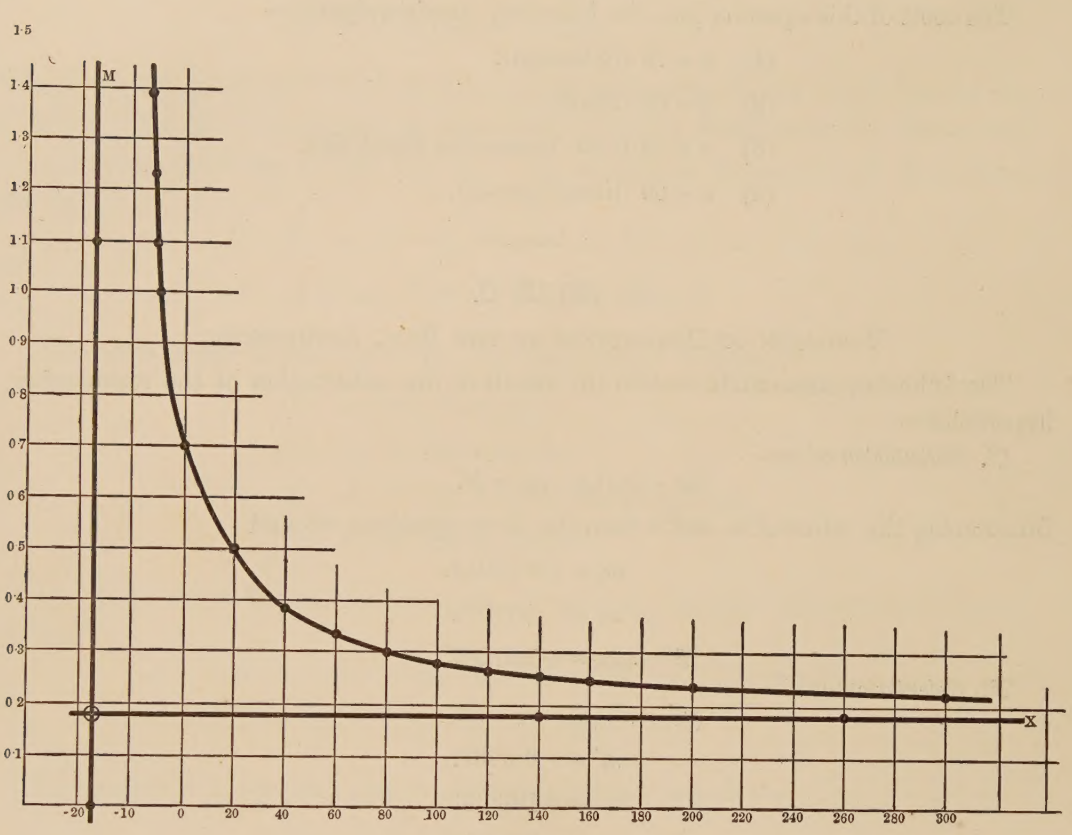



Fig. 10.

When $x$ has the value $x = -16{\cdot}671$, the value of $m$ is positive infinity, and the first asymptote makes an angle of 90° with the axis of atomic weights; this corresponds with the origin of co-ordinates of the double period, of which Hydrogen is the fourteenth element, when the manufacture of chemical atoms commenced.

When $m$ reaches the limiting value $m = +0{\cdot}18054$, the value of $x$ is positive infinity; the manufacture of chemical elements has ceased, and the sensible universe is approaching its "Nirwana," after which nothing can be save the Eternal νοῦς, which existed before matter, and will survive its death.